



1.6.8.1 Addition of complex numbers





Introduction

- In this topic, we will
 - Define the sum of two complex numbers
 - See that
 - Complex addition is commutative
 - Complex addition is associative
 - $0 = 0 + 0j$ is the additive identity





Review

- Recall from secondary the following two sums:

$$\begin{aligned}(3.2 + 4.8x) + (-5.1 + 9.6x) &= (3.2 - 5.1) + (4.8 + 9.6)x \\ &= -1.9 + 14.4x\end{aligned}$$

$$\begin{aligned}(0.5 - 4.9\sqrt{2}) + (7.3 - 1.8\sqrt{2}) &= (0.5 + 7.3) + (-4.9 - 1.8)\sqrt{2} \\ &= 7.8 - 6.7\sqrt{2}\end{aligned}$$





Complex addition

- Recall that $j \stackrel{\text{def}}{=} \sqrt{-1}$, thus if

$$z = \alpha + \beta j$$

$$w = \gamma + \delta j$$

it follows that

$$\begin{aligned} z + w &= (\alpha + \beta j) + (\gamma + \delta j) \\ &= (\alpha + \gamma) + (\beta j + \delta j) \\ &= (\alpha + \gamma) + (\beta + \delta) j \end{aligned}$$





Complex addition

- Some examples:

$$(3.2 + 4.8j) + (-5.1 + 9.6j) = (3.2 - 5.1) + (4.8 + 9.6)j \\ = -1.9 + 14.4j$$

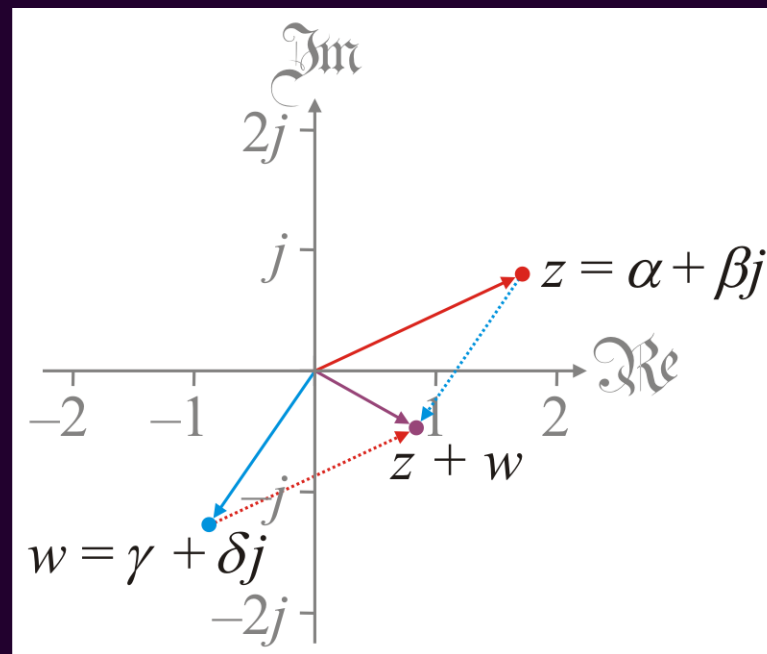
$$(0.5 - 4.9j) + (7.3 - 1.8j) = (0.5 + 7.3) + (-4.9 - 1.8)j \\ = 7.8 - 6.7j$$





Complex addition

- Going back to our geometric interpretation:





Commutativity

Theorem: Complex addition is commutative.

Proof: If $z = \alpha + \beta j$

$$w = \gamma + \delta j$$

it follows that , $z + w = (\alpha + \beta j) + (\gamma + \delta j)$

$$= (\alpha + \gamma) + (\beta + \delta)j$$

$$= (\gamma + \alpha) + (\delta + \beta)j$$

$$= (\gamma + \delta j) + (\alpha + \beta j)$$

$$= w + z \quad \text{QED}$$





Associativity

Theorem: Complex addition is associative.

Proof: If $z_1 = \alpha + \beta j$

$$z_2 = \gamma + \delta j$$

$$z_3 = \varepsilon + \zeta j$$

it follows that, $z_1 + (z_2 + z_3) = (\alpha + \beta j) + ((\gamma + \delta j) + (\varepsilon + \zeta j))$

$$\begin{aligned} &= (\alpha + \beta j) + ((\gamma + \varepsilon) + (\delta + \zeta)j) \\ &= (\alpha + (\gamma + \varepsilon)) + (\beta + (\delta + \zeta))j \\ &= ((\alpha + \gamma) + \varepsilon) + ((\beta + \delta) + \zeta)j \\ &= ((\alpha + \gamma) + (\beta + \delta))j + (\varepsilon + \zeta j) \\ &= ((\alpha + \beta j) + (\gamma + \delta j)) + (\varepsilon + \zeta j) \\ &= (z_1 + z_2) + z_3 \quad \text{QED} \end{aligned}$$





The additive identity

Theorem: $0 = 0 + 0j$ is the additive identity.

Proof: If $z = \alpha + \beta j$, then

$$\begin{aligned} z + 0 &= (\alpha + \beta j) + (0 + 0j) \\ &= (\alpha + 0) + (\beta + 0)j \\ &= \alpha + \beta j = z \quad \text{QED} \end{aligned}$$





Important takeaways

- The most important takeaways are
 - Adding complex numbers is almost as easy as adding reals
 - Given n complex numbers being added, you can add them in any order, and you'll get the same result
 - For example,
$$(3.9 + 5.7j) + (1.8 - 8.6j) + (-0.5 + 3.4j) + (1.9 - 3.1j) \\ + (3.9 + 8.4j) + (-6.4 + 1.5j) + (2.2 + 4.2j) + (4.1 - 7.0j)$$
equals $11.2 + 4.2j$





Summary

- In this topic, we've introduced complex addition
 - Very similar to polynomial addition
 - The geometric interpretation is straight-forward
 - We saw that
 - It is commutative
 - It is associative
 - $0 + 0j$ is the additive identity





References

- [1] https://en.wikipedia.org/wiki/Complex_number#Addition_and_subtraction





Acknowledgments

None so far.





Colophon

These slides were prepared using the Cambria typeface. Mathematical equations use Times New Roman, and source code is presented using Consolas.

The photographs of flowers and a monarch butter appearing on the title slide and accenting the top of each other slide were taken at the Royal Botanical Gardens in October of 2017 by Douglas Wilhelm Harder. Please see

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for more information.





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